

# Triangular Rods: geometry and engineering

Frank Stetzer

May 21, 2023

## 1 Purpose

This document shows some important geometric and engineering properties of triangular, hollow triangular, Evo13 and Evo6 (Tri-Hex) bamboo rod constructions. No doubt these derivations have been performed by others and may be available from other sources. I simply wanted to do them myself to satisfy my own curiosity.

Except for the Evo6, which I have the most interest in, there is no consideration of how these rods can be constructed by the home builder, only the properties determined by their geometrical cross sections. It is also assumed, for simplicity, that Tonkin cane is a homogeneous material. This assumption can be relaxed and I may do that at some point.

## 2 Solid triangular rods

The simplest is a solid rod equilateral in cross section. The neutral axis is one-third the height of the triangle ( $= D$ ).

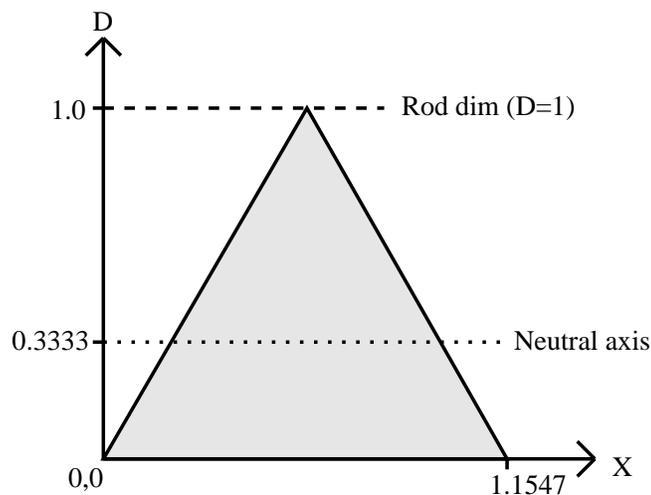


Figure 1: The triangular rod

Rod makers typically describe the size of the rod blank by its external dimension(s). I will denote the dimension by  $D$ . In all the graphs  $D$  will be set to 1 unit for simplicity, and all other dimensions and properties as functions of  $D$ .

Formulas for the area moment of inertia  $I$  are from Roark's Formulas for Stress and Strain, Table 1 section 8.

**Table 1. Properties of a solid triangular rod**

Measure	Calculate
Rod dimension	$D$
Length of external flat (base)	$B = 1.1547D$
Neutral axis	$\bar{Y} = 0.3333D$ from base
Cross Section Area	$A = \frac{1}{2}BD = 0.5774D^2$
Area moment of inertia	$I = 0.01804B^4 = 0.03207D^4$

### 3 Hollow triangular rods

The hollow rod section consists of the external triangle (Triangle 1) and the "negative" internal triangle (2). The uniform wall thickness is denoted by  $t$  and is set at 0.15 in the diagram below. For any wall thickness, the neutral axis is  $\frac{D}{3}$  from the external triangle base.

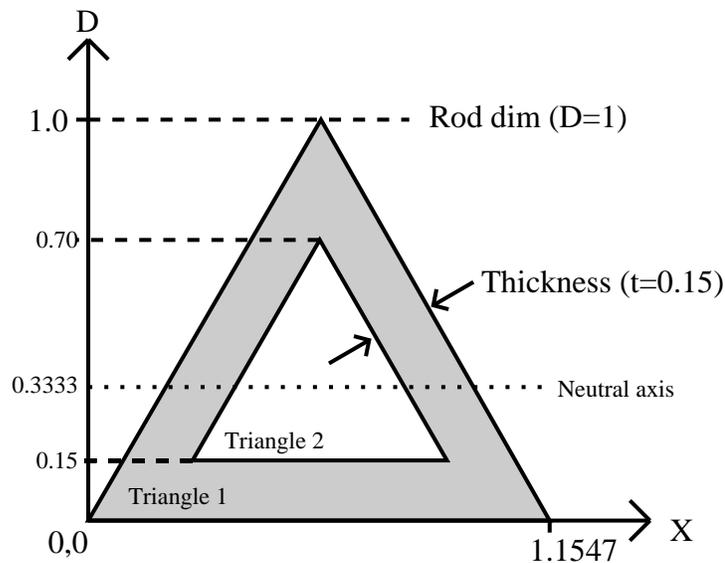


Figure 2: The hollow triangular rod

**Table 2. Properties of hollow triangular rod**

Measure	Calculate
Rod dimension	$D$
Wall thickness	$t$
Height of internal triangle	$D_2 = D - 3t$
Area of internal triangle	$A_2 = 0.5774(D - 3t)^2$
Cross sectional (bamboo) area	$A_1 - A_2 = 0.5774(6Dt - 9t^2)$
Maximum possible wall thickness	$t = \frac{D}{3}$
Neutral axis of internal triangle	$\bar{Y}_2 = t + \frac{D_2}{3} = \frac{D}{3}$
Area moment of inertia	Define: number of sides $n = 3$ Define: length of base $B = 1.1547D$ Define: $c = \frac{t \tan(60^\circ)}{B}$ $I = \frac{ntB^3}{8} \times \left(\frac{1}{3} + \frac{1}{\tan^2(60^\circ)}\right) \times (1 - 3c + 4c^2 - 2c^3)$

The formula for the moment of inertia comes from [Roark's Formulas](#) in Table 1 section 28 for hollow regular polygons.

There is an alternative approach to calculating the area moment of a structure composed of more than one “part”, some of which may be empty or negative like the hollow triangle. It is the *method of composite parts*, and requires four quantities for each of the parts: (1) whether the part is positive (existing material) or negative (hollow); (2) the moment of inertia of each; (2) the neutral axis of each; and (4) the cross section area of each. This method will be used to calculate the MoI of the Evo13 and Evo6 rod geometries in the next section.

Using the formulas in Tables 1 and 2 and the values  $D = 1$  and  $t = 0.15$ , these are the needed quantities:

**Table 3. Method of composite parts**

Measure	Outer triangle 1	Inner triangle 2
Contribution	Positive	Negative
Cross sectional area	$A_1 = 0.5774$	$A_2 = 0.2827$
Neutral axis	$\bar{Y}_1 = 0.3333$	$\bar{Y}_2 = 0.3333$
Area moment of inertia	$I_1 = 0.03207$	$I_2 = 0.00293$

The first step is to calculate the neutral axis of the combined parts. The formula is

$$\bar{Y}_c = \frac{\sum_{parts} A\bar{Y}}{\sum_{parts} A} \quad (1)$$

Parts with a negative contribution are treated as if their area is negative. This is basically the average value of the neutral axis, weighted by area of the parts.

Since both triangles have a neutral axis of 0.3333, it is obvious that their combination will have this same composite neutral axis.

$$\bar{Y}_c = \frac{0.5774 \times 0.3333 - 0.2827 \times 0.3333}{0.5774 - 0.2827} = 0.3333$$

The next step is to apply what is known as the *parallel axis theorem*. This allows the moment of inertia of each component to be adjusted so that it is in reference to a new neutral axis.

$$I_c = I + (\bar{Y} - \bar{Y}_c)^2 \times A \quad (2)$$

Again, since the combined neutral axis is identical to the original axes of the triangles, there is no adjustment to be made. The formulas become

$$\begin{aligned} I_{1c} &= 0.03207 + (0.3333 - 0.3333)^2 \times 0.5774 = 0.03207 \\ I_{2c} &= 0.00293 + (0.3333 - 0.3333)^2 \times 0.2827 = 0.00293 \end{aligned}$$

The final step is to combine these adjusted inertias, with positive parts added and negative parts subtracted.

$$I_c = I_{1c} - I_{2c} = 0.03207 - 0.00293 = 0.02914$$

Generalizing from this example with  $D = 1$  and  $t = 0.15$ , the area moment of inertia for a hollow triangular rod with dimension  $D$  and wall thickness  $t$  is

$$I_c = 0.03207(D^4 - (D - 3t)^4)$$

which is a simpler formula for  $I$  than that in Table 2.

## 4 Evo13 rods

Starting with the solid triangular rod from Figure 1, subdivide it into 16 smaller equilateral triangles as shown in Figure 3 below. Then by removing the corner triangles 1, 7, and 16, what remains is the Evo13 rod geometry cross section. In the diagram the rod dimension remains at  $D = 1$  for comparison. It can be seen that the height of the small triangles will be  $\frac{D}{3}$ .

Table 4 gives the formulas for the important Evo rod structure features. The “outer triangle” refers to the combined 16 triangles corners included. Its height is  $1.333D$ .

The area moments of inertia for Evo13 rod geometry can be calculated by the method of composite parts used above. The outer triangle is a positive contribution, and the small triangles 1, 7, and 16 are negative.

The first step is to find the composite neutral axis, following the formula in equation (1) above. Small triangles 1 and 7 have the same neutral axis so they can be combined in the calculation.

$$\begin{aligned} \bar{Y}_{Evo13} &= \frac{(0.1264D^2 \times 0.4444D) - 2(0.06415D^2 \times 0.1111D) - (0.06415D^2 \times 1.1111D)}{(1.0264 - 3 \times 0.06415)D^2} \\ &= 0.4444D \end{aligned}$$

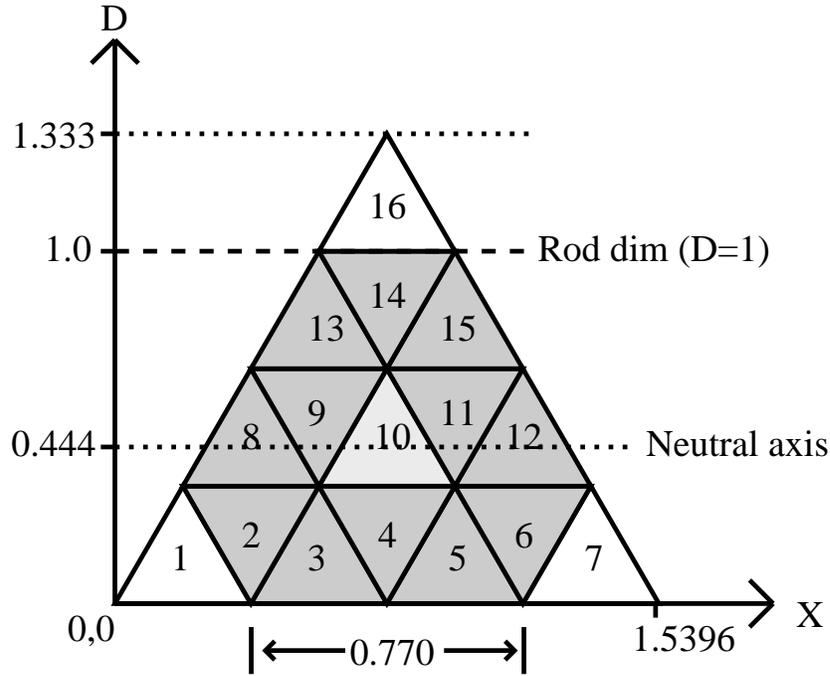


Figure 3: Evo13 and Evo6 rod geometries

The neutral axis of the original triangle and the Evo13 structure are identical.

Next, the moments of inertia of the involved triangles are adjusted to reflect the composite neutral axis by the parallel axis theorem as described in Equation (2).

**Table 5. Moments of inertia about the combined neutral axis**

Triangle	Adjusted I
Outer	$I_{outer} = 0.1013D^4 + (0.4444D - 0.4444D)^2 \times 1.0264D^2$ $= 0.1013D^4$
1 and 7	$I_1, I_7 = 0.000396D^4 + (0.1111D - 0.4444D)^2 \times 0.06415D^2$ $= 0.007522$
16	$I_{16} = 0.000396D^4 + (1.1111D - 0.4444D)^2 \times 0.06415D^2$ $= 0.02891D^4$

Finally, the moments of the four triangles are combined, with the three small triangles subtracted from the outer triangle.

$$\begin{aligned}
 I_{Evo13} &= I_{outer} - I_1 - I_7 - I_{16} \\
 &= 0.1013D^4 - 2 \times 0.007522D^4 - 0.02891D^4 \\
 &= 0.05735D^4
 \end{aligned}$$

The results are summarized in Table 4.

**Table 4. Evo13 rod structure components**

Measure	Triangle	Compute
Height	Outer	$1.3333D$
	Small	$0.3333D$
	Evo13	$D$
Base & Sides	Outer	$\frac{1.3333}{\sin 60}D = 1.5396D$
	Small	$\frac{1}{4} \times 1.5396D = 0.3849D$
	Evo13 (max width)	$3 \times 0.3849D = 1.1547D$
Cross sectional area	Outer	$A = \frac{1}{2} \times 1.3333 \times 1.5396D^2 = 1.0264D^2$
	Small	$A_s = \frac{1}{16} \times 1.0264D^2 = 0.06415D^2$
	Evo13	$A_{Evo13} = A - 3A_s = 0.8340D^2$
Neutral axis	Outer	$\bar{Y} = 0.3333 \times 1.3333D = 0.4444D$ (from base)
	Small	$\bar{Y}_{s1} = \frac{1}{3} \times 0.3333D = 0.1111D$ when base is on bottom $\triangle$ $\bar{Y}_{s2} = (\frac{1}{3} - 0.1111)D = 0.2222D$ when base is on top $\nabla$
	Evo13	$Y_{Evo13} = 0.4444D$
Moment of inertia	Outer	$I = 0.03207 \times (1.3333D)^4 = 0.1013D^4$
	Small	$I_s = 0.03207 \times (0.3333D)^4 = 0.0003958D^4$
	Evo13	$I_{Evo13} = 0.05735D^4$

## 5 Evo6 rods

### 5.1 Evo6 rods with flattened enamel sides

Hollowing the Evo13 geometry by removing triangle 10 gives the Evo6 or Tri-Hex geometry. One feature that is different from other hollow rods is that the size of the hollow, or wall thickness, is determined by the rod dimension; it is not set independently by the builder. The hollow of the Evo6 is continuously tapering from tip to butt. When the enamel faces of the involved triangles are flattened by scraping or sanding, as shown in Figure 3 for the Evo13 construction, I will refer to this as *Evo6core*.

Calculation of the moment of inertia is just an extension of the Evo13 calculation by the method of composite parts; simply subtract the additional triangle. Since triangle 10 also has a neutral axis of  $\bar{Y}_{10} = 0.4444D$ , there is no need to compute a new combined neutral axis and adjust the component moments to it.

Note that I am defining the cross sectional area (CSA) as excluding the central (hollow) triangle, as it is used when applying the method of component parts. This is in contrast to some builders who use  $A_{Evo6}$  as equal to the solid  $A_{Evo13}$ .

**Table 6. Additional Evo6 rod structure components**

Measure	Structure	Compute
Cross section area	Evo6	$A_{Evo6core} = A_{Evo13} - A_s = 0.7698D^2$
Neutral axis	Triangle 10	$\bar{Y}_{10} = \frac{1}{3} + 0.1111 = 0.4444D$
	Evo6	$\bar{Y}_{Evo6core} = 0.4444D$
Area moment of inertia	Evo6	$I_{Evo6core} = I_{Evo13} - I_{10}$
		$= 0.05735D^4 - 0.0003958D^4$
		$= 0.05695D^4$

## 5.2 Evo6 rods with unflattened enamel sides

One of the construction advantages of the Evo6 rod is that it can be planed with the same tools used for a traditional hexagonal rod (hence the name Tri-Hex.) Equilateral triangular strips are planed for the corner triangles 2, 6, and 14 of figure 3. Then three double-sized equilateral strips are planed equivalent to the combined triangles 3,4,5, and 10. The apex corresponding to triangle 10 is removed by planing or scraping, leaving three trapezoids which make the sides of the structure, with the center left hollow.

An issue with the three wide trapezoidal strips (and to a lesser degree with the small corner strips) is that, to flatten the outer surface, a circle segment of the very densest outer power fibers must be removed. Leaving the strips with their natural rounded shape will increase the strength and stiffness of the rod, in other words the area Moment of Inertia.

Figure 4 shows a greatly exaggerated view of the new structure. The additional shaded areas are called *segments* of a circle. Assign them the same number as their adjacent triangle or trapezoid. The original Evo6 structure, consisting of triangles 1, 2, and 3 and trapezoids 5, 6, and 7 will be called the *core structure*, still with dimension  $D = 1$ , and the structure with the additional outer curves the *augmented structure*. It is important to be clear that, for this paper, the dimension (e.g.  $D = 1$ ) of the Evo6 augmented rod is defined by the core structure. This will be taken up again in section 6.2.

The questions for this section are, if the sides are left unflattened, how will this alter the rod's action, and how might the rod dimensions be adjusted to compensate. It is obvious that, if we leave the core structure the same dimension, we are adding cane to the outer edge of the strips. The finished dimension of the rod will increase and the rod will become heavier and stiffer. In order to determine how much the rod will change, we need to focus on some specific measures. I will look at two: the cross sectional area (CSA) for a measure of size and weight, and the area moment of inertia (MoI), a measure of the stiffness of the structure. More complex measures such as stress and deflection I may investigate in the future.

We will start with some items needed for calculations. Figure 5(a) is from Roark's Formulas Table 1 section 19. Figure 5(b) shows the final augmented equilateral strip which will result from planing. The height of the triangular part of strip  $\triangle ABG$  is  $h = \overline{FG}$  which is  $h = \frac{1}{3}D$  for the narrow planed triangular strips (1, 2, and 3) or  $h = \frac{2}{3}D$  for the wide strips (5, 6, and 7). The height of augmented part is  $e = \overline{EF}$ , which can be determined along with other quantities once the angle  $\alpha$  is determined.

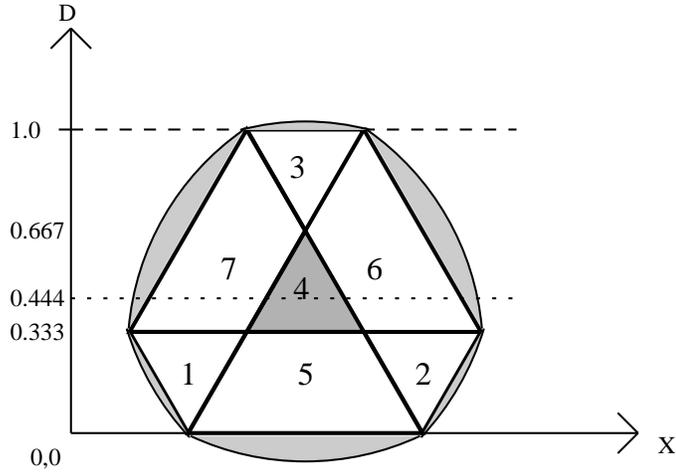


Figure 4: Evo6 geometry showing rounded strip outer surfaces

Starting with 5(a), the radius of the circle, which is the radius of the bamboo culm, is  $R$ . The subtended angle to the center of the culm cross section is  $\theta = 2\alpha$ , which will be in radians for the calculations that follow. 1–1 and 2–2 are the principal axes of the segment; when oriented as shown (i.e. attached to triangle 3), axis 1–1 is the neutral axis and the intersection of 1–1 and 2–2 is the segment centroid. In this orientation, the moment of inertia as part of the rod is at its maximum and will be less when rotated to another orientation. Table 7 gives formulas for the basic components of the segment, from Roark's.

**Table 7. Circle segment components**

Measure	Roark's formula
Cross Section Area	$A = \frac{2}{3}R^2\alpha^3(1 - 0.2\alpha^2 + 0.019\alpha^4)$
Axis 1 from outside	$y_{1a} = 0.3R\alpha^2(1 - 0.0976\alpha^2 + 0.0028\alpha^4)$
Axis 1 from inside	$y_{1b} = 0.2R\alpha^2(1 - 0.0619\alpha^2 + 0.0027\alpha^4)$
Axis 2 from corner	$y_2 = R\alpha(1 - 0.1667\alpha^2 + 0.0083\alpha^4)$
Moment of inertia wrt Axis 1	$I_1 = 0.01143R^4\alpha^7(1 - 0.3491\alpha^2 + 0.0450\alpha^4)$
Moment of inertia wrt Axis 2	$I_2 = 0.1333R^4\alpha^5(1 - 0.4762\alpha^2 + 0.1111\alpha^4)$

In order to utilize these formulas it is necessary to determine  $\alpha$  from the two initial values culm radius  $R$  and rod dimension  $D$ . The first step is to determine the length of the chord  $\overline{AB}$ , which is the length of a side of the equilateral triangle. This length is  $s = \frac{h}{\cos 30^\circ}$ . Substituting the formula(s) for  $h$  as a function of  $D$  results in  $s = 0.3849D$  for the narrow strips and  $s = 0.7698D$  for the wide. The formula for the subtended angle  $\theta$  (in radians) is  $\theta = 2 \arcsin(\frac{s}{2R})$  and  $\alpha = \frac{1}{2}\theta$ .

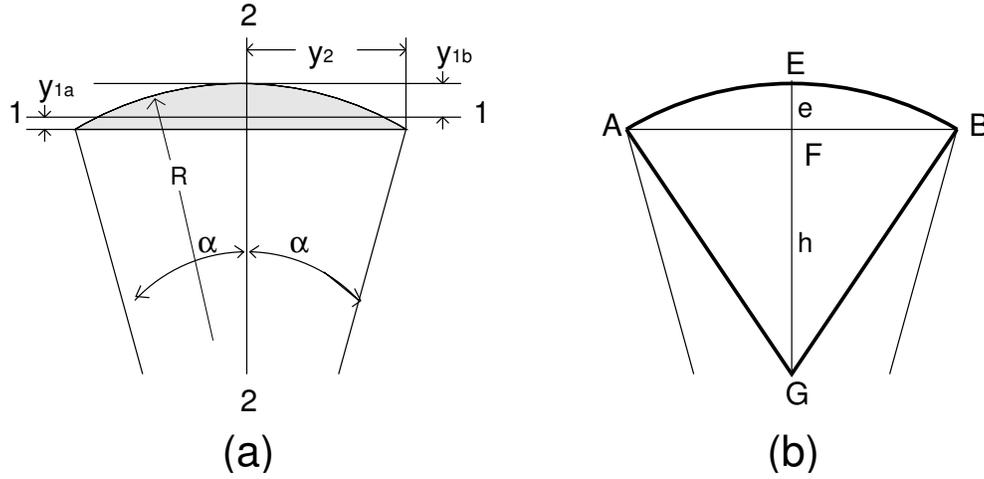


Figure 5: Components of the segment of a circle

The final equations for  $\alpha$  for the narrow and wide strips become

$$\text{for narrow strips} \quad \alpha = \arcsin\left(\frac{0.3849D}{2R}\right) \quad (3)$$

$$\text{for wide strips} \quad \alpha = \arcsin\left(\frac{0.7698D}{2R}\right) \quad (4)$$

With  $\alpha$  and  $R$  the circle segment components in Table 7 can be computed. The height of the augmented area will be  $e = y_{1a} + y_{1b}$ .

Before calculating the values of  $y_{1a}$  and  $y_{1b}$  from Table 7, it is insightful to dissect the equations a little. The for the relatively small values of  $\alpha$  we are dealing with, the second term  $(1 - c_1\alpha^2 + c_2\alpha^4)$  is very close to 1.0, typically 0.999 to three decimal places. If we ignore this multiplier, the total height of the augmented section is

$$e = y_{1a} + y_{1b} \approx 0.5R\alpha^2 \quad (5)$$

While the angle  $\alpha$  doubles for the wide strips compared to narrow, the total height of the augmented addition is 4 times larger in the wide strips. This simplification can be used in the calculations without loss of practical precision.

The additional CSA of the augmented structure is calculated directly from Roark's formula in Table 7. In order to calculate the moment of inertia of the augmented structure, the method of component parts will again be used. The moments of the six augmented circle segments can be combined with the moment of the core, as given in Table 6. But first the moments  $I_1$  and  $I_2$  of the generic segment shown in Figure 5 must be adjusted for the six strip rotations pictured in Figure 4, and the neutral axis of the rotated segments calculated. This can be accomplished by a computational tool known as *Mohr's Circle* which can be found in mechanical engineering texts or online. The neutral axis of the augmented Evo6 structure remains at  $0.4444D$ , which simplifies the calculations.

Tables 8A and 8B show what happens to CSA and MoI when the structure changes from flattened enamel to augmented, assuming a typical culm size of R=1.25 inches. The first two columns give the dimensions and CSA/MoI assuming flattened enamel; the next two assume the enamel is not flattened while the rod core dimension remains the same. The change column gives the relative increase in CSA/MoI with the addition of the outer augmentation. Logically this change becomes relatively larger as rod dimension increases, and the affect on MoI is larger than on CSA. If we would build two rods with same core dimensions, one with flattened enamel and the other augmented, the later would be considerably stiffer in the butt region. The final two columns show what the rod dimensions would be, both core and augmented, to arrive at a rod with the same CSA or MoI as the original flattened rod. These results are only for the specified culm radius of 1.25 inches. Intuitively, the smaller the culm radius, the more "curvature" to the outer surface of the augmented strips, and the greater the effect on CSA and MoI.

**Table 8A. Affect of Evo6 augmentation on CSA, R=1.25**

Flattened		Augmented			Dim to make equal	
$D_{core}$	CSA	$D_{aug}$	CSA	CSA Chg	$D_{core}$	$D_{aug}$
0.050	0.001924	0.0502	0.001937	+0.67%	0.0499	0.0500
0.100	0.007698	0.1007	0.007801	+1.33%	0.0994	0.1000
0.150	0.01732	0.1517	0.01767	+2.00%	0.1487	0.1500
0.200	0.03079	0.2030	0.03161	+2.67%	0.1977	0.2000
0.250	0.04811	0.2546	0.04972	+3.34%	0.2464	0.2500
0.300	0.06928	0.3067	0.07206	+4.01%	0.2954	0.3000
0.350	0.09430	0.3591	0.09872	+4.68%	0.3413	0.3501

**Table 8B. Affect of Evo6 augmentation on MoI, R=1.25**

Flattened		Augmented			Dim to make equal	
$D_{core}$	MoI	$D_{aug}$	MoI	MoI Chg	$D_{core}$	$D_{aug}$
0.050	3.593e-7	0.0502	3.559e-7	+0.95%	0.0499	0.0501
0.100	5.695e-6	0.1007	5.804e-6	+1.91%	0.0995	0.1003
0.150	2.883e-5	0.1517	2.966e-5	+2.89%	0.1489	0.1506
0.200	9.112e-5	0.2030	9.465e-5	+3.87%	0.1981	0.2010
0.250	2.224e-4	0.2536	2.333e-4	+4.87%	0.2471	0.2516
0.300	4.613e-4	0.3067	4.884e-4	+5.88%	0.2958	0.3023
0.350	8.546e-4	0.3591	9.137e-4	+6.91%	0.3443	0.3531

A note on the calculation of the last two columns. There might be an equation or set of equations that will give these results, but I did not seek it. I solved it by an iterative process that involved finding the ratio of flattened CSA (or MoI) to augmented, and using this to calculate an adjustment factor for the sizes of the triangles involved, the resulting angles and the components of Table 7. Two iterations were all that was required to get practical convergence.

## 6 Taper conversion

### 6.1 Conversion formulas

In this final section I will give the formulas for converting a taper from a traditional solid flat-sided hexagonal rod to the three Evo rods: Evo13, Evo6 core and Evo6 augmented. There is no reason that tapers for a new rod geometry needs to be derived from an existing tapers, but it is often a convenient place to start. A new rod geometry will, hopefully, have unique characteristics that can be taken advantage of by new tapers.

Two conversion methods are equal cross sectional area, which is equivalent to equal weight at each point along the taper, and equal moment of inertia, which is equivalent to equal stiffness and, therefore, equal static deflection. Other methods such as equal stress or casting deflection are not considered here but are possible.

The basic method is simple: equate the formula for cross sectional area (or MoI) for a hexagonal rod structure of dimension  $D_{hex}$  to that for a Evo rod of dimension e.g.  $D_{Evo6core}$ ; then solve the equation for  $D_{Evo6core}$  in terms of  $D_{hex}$ . The user then can take dimension values from the hexagonal rod and use this equation to compute the dimension value for a triangular rod. The same general approach works between other rod structures. Here is an example for cross sectional area that illustrates the idea:

$$\begin{aligned} 0.8340D_{Evo13}^2 &= 0.8660D_{hex}^2 \\ D_{Evo13} &= \sqrt{\frac{0.8660}{0.8340}}D_{hex} \\ &= 1.019D_{hex} \end{aligned}$$

To convert from Hex to Evo6 augmented, the first step is to convert to the Evo6 core dimensions. For example, consider the formula to convert from a hex to an Evo6 core taper by equal MoI:

$$\begin{aligned} 0.05695D_{Evo6core}^4 &= 0.06014D_{hex}^4 \\ D_{Evo6core} &= \sqrt[4]{\frac{0.06014}{0.05695}}D_{hex} \\ &= 1.01372D_{hex} \end{aligned}$$

Then to get the augmented rod dimensions, the same iterative process that was used to calculate the values in the “Dim to make equal” columns of Tables 8A and 8B needs to be applied. If you notice the near-equivalence of values in the first  $D_{core}$  and last  $D_{aug}$  columns, however, you would be justified in skipping this step and simply using the Evo6 core formula above, unless your culms are unusually small or your rod dimensions large (e.g. a two-handed rod). This might not hold for translation from a quad or penta taper to triangular; I have not tried it. In the next section on planing form settings more details emerge, however. This result is assumed in Table 9 below.

**Table 9. Conversion formulas from hex dimensions**

Equivalent	Formula	
Cross section area	Evo 13	$D_{Evo13} = 1.0190D_{hex}$
	Evo 6 core	$D_{Evo6core} = 1.0606D_{hex}$
	Evo 6 augmented	$D_{Evo6aug} = 1.0606D_{hex}$ for typical size culm
Moment of Inertia	Evo 13	$D_{Evo13} = 1.0120D_{hex}$
	Evo 6 core	$D_{Evo6core} = 1.0137D_{hex}$
	Evo 6 augmented	$D_{Evo6aug} = 1.0137D_{hex}$ for typical size culm

## 6.2 Planing form settings

Planing form settings for Evo13 and Evo6 core structures, in other words those with flattened enamel surfaces, are straightforward. For Evo13 they are  $\frac{1}{3}D_{Evo13}$ , and for Evo6 core the narrow strips for the corners are also  $\frac{1}{3}D_{Evo6core}$  and the wide strips which are destined to be trapezoids are  $\frac{2}{3}D_{Evo6core}$ .

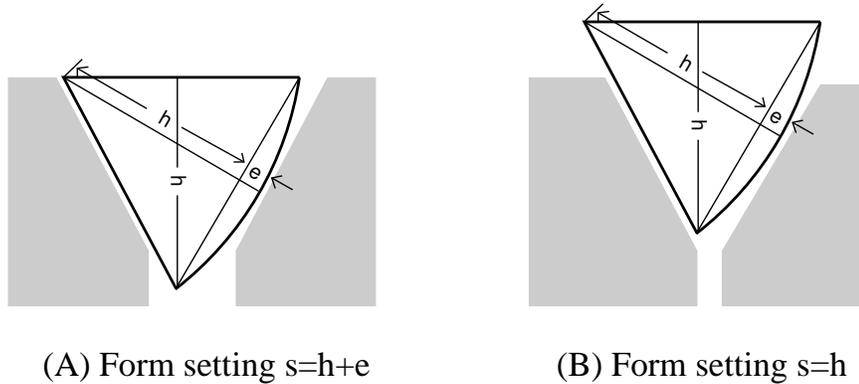


Figure 6: Planing form settings for unflattened surface

For the Evo6 augmented structure, things become a bit more complicated, as can be seen in Figure 6. (The curved culm surface is greatly exaggerated for illustration.) If we set the form to the Evo6 core numbers from Table 9, the situation is as seen in the right. While part of the outer power fibers will be included in the final planed strip, some will be planed away, along with part of core equilateral triangle. To determine the correct setting, start with  $\frac{1}{2}$  and  $\frac{2}{3}$  of the  $D_{Evo6core}$  value, and then add the height of the augmented region  $e = y_{1a} + y_{1b}$  from Table 7 above; thus it depends on the angle  $\alpha$  which itself depends on  $R$  and  $D_{Evo6}$  as seen in equations (3) and (4). In other words, the iterative process described on page 10 needs to be used. If desired, calculation of  $e$  can be shortened to  $e = 0.5R\alpha^2$  without loss of practical precision.

Table 9 gives values of  $e$  for different core strip heights. Depending on how you are arriving at your taper, these might be useful for setting your forms to allow for the augmentation.

**Table 9. Augmented section height  $e$**

<b>Core Strip Height</b>	<b>Culm Radius</b>				
	<b>0.75</b>	<b>1.0</b>	<b>1.25</b>	<b>1.5</b>	<b>1.75</b>
0.025	0.0001	0.0001	0.0001	0.0001	0.0001
0.050	0.0006	0.0004	0.0003	0.0003	0.0002
0.075	0.0012	0.0009	0.0008	0.0006	0.0005
0.100	0.0022	0.0017	0.0013	0.0011	0.0010
0.125	0.0035	0.0026	0.0021	0.0017	0.0015
0.150	0.0050	0.0037	0.0030	0.0025	0.0021
0.175	0.0068	0.0051	0.0041	0.0034	0.0029
0.200	0.0089	0.0067	0.0053	0.0044	0.0038